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# Triplet Josephson current modulated by Rashba spin–orbit coupling

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## Abstract

We study the Rashba spin–orbit coupling (RSOC) effect on the supercurrent in a clean triplet superconductor/two-dimensional electron gas/triplet superconductor (TS/2DEG/TS) junction, where RSOC is considered in the 2DEG region. Based on the Bogoliubov–de Gennes equation and quantum scattering method, we show that RSOC can lead to a  $0-\pi$  oscillation of supercurrent and the abrupt current reversal effect. The current direction can be reversed by a tiny modulation of RSOC, and this is attributed to the equal spin pairing of the TS order parameter and the spin precession phase of the quasiparticle traveling in the RSOC region. The RSOC strength can be controlled by an electric field in experiments, thus our findings provide a purely electric means to modulate the supercurrent in TS Josephson junctions.

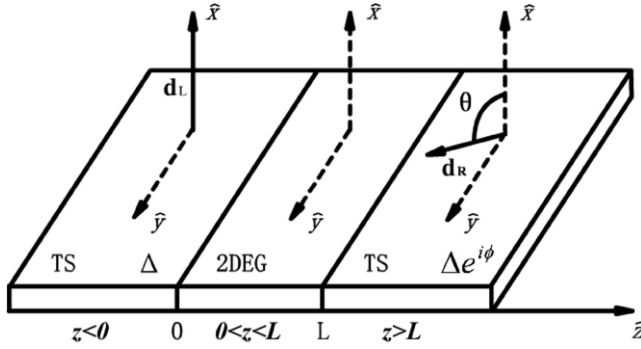
(Some figures in this article are in colour only in the electronic version)

## 1. Introduction

The Josephson junction is an active research field in condensed-matter physics, since it is a basic building block for superconducting electronics with applications that range from SQUID magnetometers to possible quantum computing devices [1–3]. Modulation of the supercurrent is important for various potential applications of Josephson junctions, and a great deal of work has been devoted to this research field. Recently, the so-called  $0-\pi$  state transition in conventional SC/FM/SC junctions (FM, ferromagnetic metal) has attracted much attention [4–8]. The  $\pi$  state is referred to as the ground state of the junction at the macroscopic phase difference  $\phi = \pi$ , and the critical current direction is opposite to the 0 state. The  $\pi$  state in the SC/FM/SC junction is attributed to the tunneling Cooper pair possessing a nonzero momentum due to FM exchange splitting in the FM region. Actually, it is not very convenient to modulate the FM layer length or the FM exchange strength in the SC/FM/SC junction to realize the  $0-\pi$  transition in a single device. Moreover, ferromagnetism is unfavorable for the spin-singlet order parameter in the FM region, and the critical current is suppressed greatly by FM. Therefore, other alternatives [9–13], such as spin–orbit interaction in two-dimensional electron gases (2DEGs), were proposed to replace FM in realizing the  $\pi$  state and controlling

the supercurrent. A spin–orbit interaction in semiconductors is much more desirable, because it is easy to integrate into devices, and makes pure electric manipulation of devices possible, without using any FM element or magnetic field.

Many authors [10–12] have studied the Rashba spin–orbit coupling (RSOC) effect on the s-wave SC/2DEG/SC junction (2DEG: two-dimensional electron gas), and they concluded RSOC can not modulate the supercurrent as can FM. It is agreed that RSOC keeps time reversal symmetry and the singlet Cooper pair can not achieve any associated phase, when the order parameter enters into the RSOC region, especially for the one-dimensional case. This reminds us of the case of the p-wave SC junction. The triplet order parameter is expected to achieve a spin precession phase when the tunneling Cooper pairs travel in the RSOC region, thus it is desirable to study the RSOC effect on the triplet superconductor (TS) junction. The Cooper pair potential of a TS is anisotropic and  $\mathbf{k}$ -dependent,  $\Delta(\mathbf{k}) \sim i\mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma} \sigma_y$ , where  $\mathbf{d}$  is a vector characterizing the TS Cooper pair and  $\boldsymbol{\sigma}$  is a Pauli operator with three components,  $\sigma_{x,y,z}$ . Therefore, a spin supercurrent [14–19] may flow through the junction besides the usual charge supercurrent, this spin supercurrent results from the misalignment of  $\mathbf{d}$  vectors in two triplet SC leads. Although the vector  $\mathbf{d}$  appears like a magnetic moment, it is nonobservable and preserves time reversal symmetry in a unitary state  $\mathbf{d} \times \mathbf{d}^* = 0$ . Recently,



**Figure 1.** Schematic diagram of the TS/2DEG/TS junction studied in this work, the current direction is along the  $z$ -axis and the  $\mathbf{d}$  vectors are in the  $xy$  plane.

Kastening *et al* [14, 15] found that unparallel  $\mathbf{d}$  vectors (with cross angle  $\theta$ ) in a TS Josephson junction can have a huge effect on the supercurrent, which exhibits an abrupt current sign reversal or current switch effect at suitable  $\theta$ . This finding is very useful in the field of superconductor electronics and quantum information technology. However, it is a big challenge to modulate the cross angle  $\theta$ , since the direction of  $\mathbf{d}$  is actually determined by the crystal axis. Therefore, it is desirable to find some alternative to realize the abrupt current reversal effect and control the supercurrent.

In this work, we study the RSOC effect on a clean TS Josephson junction, which is composed of two equal spin pairing TS leads and a 2DEG between them, with the RSOC merely considered in the 2DEG region. A quantum scattering method based on the Bogoliubov–de Gennes (BdG) equation is employed to calculate the Josephson current. It is found that RSOC has a significant effect on the supercurrent: firstly, RSOC can indeed modulate the supercurrent, and the  $0-\pi$  transition of the TS junction is readily realized; secondly, the length of the RSOC region is also a parameter to control the  $0-\pi$  transition, since the spin precession phase from RSOC is determined by both the RSOC strength and the length of the 2DEG layer; finally, the abrupt current reversal effect found in [14, 15] is also achievable by merely modulating the RSOC, which is more conveniently modulated than the direction of  $\mathbf{d}$  or the FM moment. As a matter of fact, the strength of RSOC was shown in experiments by Nitta *et al* [20] to be modulated by an electric field perpendicular to the 2DEG plane.

The organization of this paper is as follows. In section 2, we present the model and formulae to calculate the Josephson current. In section 3 we discuss the dependence of Josephson current on the several parameters characterizing the system, especially the RSOC. A brief conclusion is drawn in section 4.

## 2. Model and formulation

We consider a clean TS/2DEG/TS Josephson junction as shown schematically in figure 1, constructed by sandwiching a 2DEG layer with the RSOC effect between two semi-infinite TS leads. The left (L) and right (R) TS are assumed identical except for the different macroscopic phases and  $\mathbf{d}$  vectors. Two interface barriers at  $z = 0$  and  $z = L$  are in the  $xy$  plane, and

the quantum spin axis is set along the  $z$ -axis. The BdG equation describing the TS junction is given by [21]

$$\begin{pmatrix} H(\mathbf{k}) & \Delta(\mathbf{k}) \\ -\Delta^\dagger(-\mathbf{k}) & -H^*(-\mathbf{k}) \end{pmatrix} \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix} = E \begin{pmatrix} \hat{u} \\ \hat{v} \end{pmatrix}, \quad (1)$$

where  $H(\mathbf{k}) = H_0(\mathbf{k}) + H_{\text{RSOC}}(\mathbf{k})$ ,  $H_0(\mathbf{k}) = \varepsilon_{\mathbf{k}} - \mu + U$ .  $\mu$  is the chemical potential of the system, and the interface potential is modeled by  $U = U_0[\delta(z) + \delta(z - L)]$ .  $H_{\text{RSOC}} = \frac{\lambda}{\hbar}(\sigma_y p_z - \sigma_z p_y)\theta_0(z)$  describes RSOC in the 2DEG region [22] with  $\lambda$  the RSOC constant, and  $p_y, p_z$  are the two components of the momentum operator  $\mathbf{p}$ ,  $\theta_0(z) = \theta(z)\theta(-z+L)$  with  $\theta(z)$  the Heaviside step function. Neglecting the self-consistency of the superconducting pair potential,  $\Delta(\mathbf{k})$  is taken in the form

$$\Delta_j(\mathbf{k}) = i\Delta_0(\mathbf{d}_j(\mathbf{k}) \cdot \boldsymbol{\sigma})\sigma_y e^{i\phi_j}, \quad (2)$$

where  $\Delta_0$  is a constant.  $\phi_{j=L(R)}$  is the macroscopic phase of SC lead,  $\phi_L = 0$  and  $\phi_R = \phi$  are set in our consideration without loss of generalization. The  $\mathbf{d}_j$  vector is in the  $xy$  plane and  $\mathbf{d}_j = (\cos(\theta_j), \sin(\theta_j), 0)$ .  $\mathbf{d}_L$  is parallel to the  $x$  axis ( $\theta_L = 0$ ) and  $\mathbf{d}_R$  has an arbitrary angle  $\theta$  upon the  $x$ -axis as shown in figure 1. The  $p_z$  orbital symmetry of the pair potential in both SC leads is taken into account,  $\Delta(k_z) = -\Delta(-k_z)$  the same as that in [14, 15].

The Josephson current can be expressed in terms of the Andreev reflection (AR) coefficients by using the Furusaki–Tsukada formalism [23]

$$I_J = \frac{e\Delta_0}{\hbar} \sum_{k_\parallel} \int \frac{dE}{2\pi} \left[ \frac{(k_e + k_h)}{2\Omega} \left( \frac{c_1}{k_e} + \frac{d_2}{k_e} - \frac{a_3}{k_h} - \frac{b_4}{k_h} \right) + \text{c.c.} \right] f(E), \quad (3)$$

where  $c_1, d_2$  are two spin-resolved AR coefficients of electron-like quasiparticle, while  $a_3, b_4$  are the AR coefficients of hole-like quasiparticles of the two spin species.  $k_e, k_h$  are respectively the  $z$ -component wavevectors of the electron-like and hole-like quasiparticles in the TS leads, and  $k_\parallel$  is their transverse momentum,  $\Omega = \sqrt{E^2 - \Delta_0^2}$  with quasiparticle energy  $E$ .

To obtain the four Andreev reflection coefficients, one needs to determine the scattering waves in different regions for each scattering process, e.g., the process for the spin-up electron-like quasiparticle entering the 2DEG region from the left SC lead and scattering at both interfaces is given by

$$\begin{aligned} \psi_{e1}(z \leq 0) = e^{ik_\parallel y} & \left[ \begin{pmatrix} ue^{-i\theta_{L+}} \\ 0 \\ v \\ 0 \end{pmatrix} e^{ik_e z} + a_1 \begin{pmatrix} ue^{-i\theta_{L-}} \\ 0 \\ v \\ 0 \end{pmatrix} e^{-ik_e z} \right. \\ & + b_1 \begin{pmatrix} 0 \\ -ue^{i\theta_{L-}} \\ 0 \\ v \end{pmatrix} e^{-ik_e z} + c_1 \begin{pmatrix} ve^{-i\theta_{L+}} \\ 0 \\ u \\ 0 \end{pmatrix} e^{ik_h z} \\ & \left. + d_1 \begin{pmatrix} 0 \\ -ve^{i\theta_{L+}} \\ 0 \\ u \end{pmatrix} e^{ik_h z} \right], \quad (4) \end{aligned}$$

$$\begin{aligned} \psi_{e1}(z \geq L) = & e^{ik_{\parallel}y} \left[ e_1 \begin{pmatrix} ue^{-i(\theta_{R+}-\phi)} \\ 0 \\ v \\ 0 \end{pmatrix} e^{ik_{e2}z} \right. \\ & + f_1 \begin{pmatrix} 0 \\ -ue^{i(\theta_{R+}+\phi)} \\ 0 \\ v \end{pmatrix} e^{ik_{e2}z} + g_1 \begin{pmatrix} ve^{-i(\theta_{R-}-\phi)} \\ 0 \\ u \\ 0 \end{pmatrix} e^{-ik_{h2}z} \\ & \left. + h_1 \begin{pmatrix} 0 \\ -ve^{i(\theta_{R-}+\phi)} \\ 0 \\ u \end{pmatrix} e^{-ik_{h2}z} \right], \end{aligned} \quad (5)$$

and

$$\begin{aligned} \psi_{e1}(0 < z < L) = & e^{ik_{\parallel}y} \left[ \frac{a_{m1}}{\sqrt{2}} \begin{pmatrix} e^{i\chi(\alpha_1)} \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{iq_{e1}z} + \frac{b_{m1}}{\sqrt{2}} \begin{pmatrix} -e^{i\chi(\alpha_2)} \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{iq_{e2}z} \right. \\ & + \frac{c_{m1}}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ -e^{-i\chi(\alpha_{1r})} \\ 1 \end{pmatrix} e^{-iq_{h1}z} + \frac{d_{m1}}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ e^{-i\chi(\alpha_{2r})} \\ 1 \end{pmatrix} e^{-iq_{h2}z} \\ & + \frac{e_{m1}}{\sqrt{2}} \begin{pmatrix} e^{i\chi(\alpha_{1r})} \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-iq_{e1}z} + \frac{f_{m1}}{\sqrt{2}} \begin{pmatrix} -e^{i\chi(\alpha_{2r})} \\ 1 \\ 0 \\ 0 \end{pmatrix} e^{-iq_{e2}z} \\ & \left. + \frac{g_{m1}}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ -e^{-i\chi(\alpha_1)} \\ 1 \end{pmatrix} e^{iq_{h1}z} + \frac{h_{m1}}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ e^{-i\chi(\alpha_2)} \\ 1 \end{pmatrix} e^{iq_{h2}z} \right], \end{aligned} \quad (6)$$

where

$$u = \sqrt{\frac{1}{2} + \frac{\Omega}{2E}}, \quad v = \sqrt{\frac{1}{2} - \frac{\Omega}{2E}}, \quad (7)$$

$\theta_{j-} = \theta_{j+} + \pi$ , the scattering coefficients  $a_1, b_1$  are two spin-resolved normal reflections, while  $c_1, d_1$  are the ARs in equation (4).  $q_{e1}, q_{e2}, q_{h1}$ , and  $q_{h2}$  are the spin-dependent  $z$ -component wavevector of electrons and holes in the 2DEG region.  $\chi(\alpha_i) = \pi/2 - \alpha_i$ ,  $\alpha_{ir} = \pi - \alpha_i$  and  $\alpha_{i=1(2)}$  are the angles between particle wavevectors and the  $z$ -axis. Note that the wavefunctions in equation (6) are presented with the assumption of the spin quantum axis set along the normal of the 2DEG plane, and they can be readily transferred to the case of the spin quantum axis along the  $z$  direction by a unitary transformation. The wavevectors of particles in these three regions are expressed as  $k_{e0} = \sqrt{k_F^2 + 2m\Omega/\hbar^2}$ ,  $k_{h0} = \sqrt{k_F^2 - 2m\Omega/\hbar^2}$ ,  $q_{e10} = q_{h10} = \sqrt{k_R^2 + k_F^2 - k_R}$ , and  $q_{e20} = q_{h20} = \sqrt{k_R^2 + k_F^2 + k_R}$ , with  $k_F = \sqrt{2mE_F/\hbar^2}$  and  $k_R = m\lambda/\hbar^2$  being respectively the Fermi wavevector and Rashba wavevector,  $E_F$  is the Fermi energy. Since translational symmetry is conserved along the interface direction,  $k_e = \sqrt{k_{e0}^2 - k_{\parallel}^2}$ ,  $k_h = \sqrt{k_{h0}^2 - k_{\parallel}^2}$ ,  $q_{e1} = q_{h1} = \sqrt{q_{e10}^2 - k_{\parallel}^2}$ , and

$q_{e2} = q_{h2} = \sqrt{q_{e20}^2 - k_{\parallel}^2}$ , with  $k_{\parallel}$  the transverse momentum parallel to the interface. The dimensionless parameter  $Z = 2mU_0/\hbar^2k_F$  is defined as a measurement of the insulator barrier strength and  $\beta = 2k_R/k_F$  represents the strength of RSOC in the 2DEG region. The 16 coefficients in equations (4)–(6) can be determined by applying the boundary conditions [24, 25] to the wavefunctions at  $z = 0$  and  $z = L$  as follows:

$$\begin{aligned} \psi(z)|_{z=+0} &= \psi(z)|_{z=-0}, \\ v_z \psi(z)|_{z=+0} - v_z \psi(z)|_{z=-0} &= Z_0 \tau_3 \psi(0), \\ \psi(z)|_{z=L+0} &= \psi(z)|_{z=L-0}, \\ v_z \psi(z)|_{z=L+0} - v_z \psi(z)|_{z=L-0} &= Z_0 \tau_3 \psi(L), \end{aligned} \quad (8)$$

with  $Z_0 = Zk_F/im$  and

$$\tau_3 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The velocity operator in the 2DEG lead along the  $z$  direction in the boundary conditions is given by

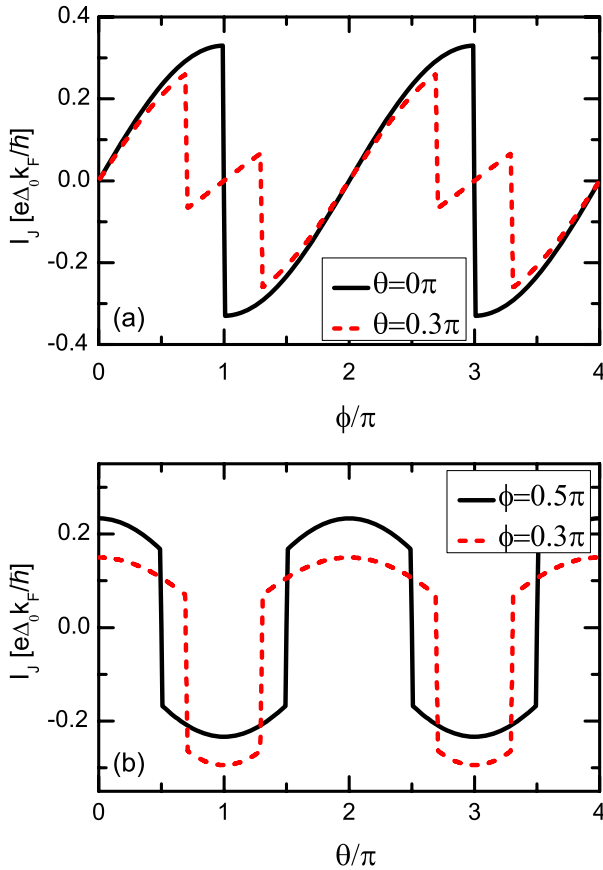
$$v_{jz} = \frac{\partial H}{\hbar \partial k_z} = \begin{pmatrix} \frac{\hbar}{im_j} \frac{\partial}{\partial z} & \frac{i\lambda}{\hbar} \theta_0(z) & 0 & 0 \\ -\frac{i\lambda}{\hbar} \theta_0(z) & \frac{\hbar}{im_j} \frac{\partial}{\partial z} & 0 & 0 \\ 0 & 0 & -\frac{\hbar}{im_j} \frac{\partial}{\partial z} & -\frac{i\lambda}{\hbar} \theta_0(z) \\ 0 & 0 & \frac{i\lambda}{\hbar} \theta_0(z) & -\frac{\hbar}{im_j} \frac{\partial}{\partial z} \end{pmatrix}. \quad (9)$$

By solving the 16 linear equations, one can obtain the useful AR coefficient  $c_1$  in equation (4). Certainly, there are three other scattering processes needed to determine the corresponding AR coefficients,  $d_2, a_3, b_4$ , which together with  $c_1$  are input into equation (3) for computing the Josephson current.

### 3. Results

In this section, we present the numerical results of the Josephson current flowing in the TS/2DEG/TS junction at zero temperature  $T = 0$  K. The Fermi wavevectors are set the same in all three regions for simplicity, and the effect of a mismatch of Fermi wavevector is actually equivalent to varying the interface barrier strength. To simplify our calculations, only the one-dimensional case is focused on in this work, as in [14, 15], by assuming zero transverse momentum  $k_{\parallel} = 0$ . The Josephson current in the 2D case should not be qualitatively different from the 1D case, moreover, the 1D RSOC has been shown not to have an effect on the conventional s-wave Josephson junction. Therefore, one needs firstly to check whether the 1D RSOC can make a difference to the p-wave Josephson junction.

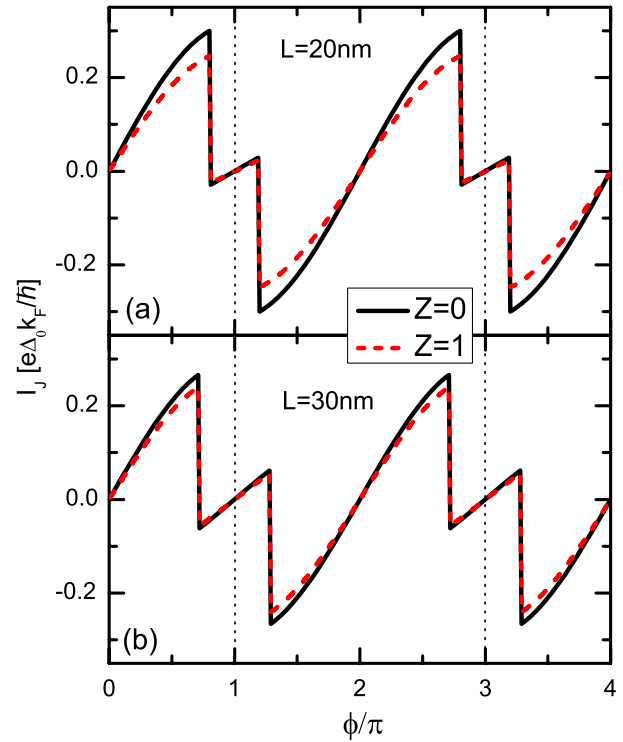
As stated earlier, an abrupt current reversal effect occurs in the TS/2DEG/TS junction, i.e., the current direction can be switched swiftly by a tiny variation of the cross angle  $\theta$  at the suitable parameters. We first focus on the non-RSOC case ( $\beta = 0$ ) and present the supercurrent  $I_J$  as a function of  $\phi$  and  $\theta$  in figure 2. The supercurrent in figure 2(a) has a discontinuous jump at  $\phi_{ZC} \equiv (2n + 1)\pi$  with  $\theta = 0$  (solid line). This is



**Figure 2.** The dependence of the Josephson current on the macroscopic phase  $\phi$  (a) and the cross angle  $\theta$  (b),  $\beta = 0$ ,  $L = 20$  nm,  $Z = 0$ ,  $k_F = 6.0 \times 10^8$  m<sup>-1</sup>.

the same as the supercurrent in the usual s-wave SC junction because the Andreev bound states in energy gap are degenerate at  $E = 0$  when  $\phi = \phi_{ZC}$ . In the s-wave case, the discontinuous jump can be smeared by an insulator barrier, whereas it is robust in p-wave junction due to the edge states [26, 27] at the junction interface. When  $\theta \neq n\pi$ , the two  $\mathbf{d}$  vectors in the TS leads are not parallel or antiparallel. The discontinuous jump is split into two steps at  $\phi_{ZC} \pm \theta$  (the dashed line in figure 2(a)), because the Andreev bound states' degeneration ( $E = 0$ ) occurs at  $\phi_{ZC} \pm \theta$ . The charge supercurrent is now spin-split due to the misalignment of  $\mathbf{d}$  vectors, which actually accounts for the spin supercurrent flowing [14, 15] in this TS junction. This interesting phenomena is shown in figure 2(b). The  $I_J$  can be modulated by the cross angle  $\theta$  and the current direction is abruptly reversed at  $\theta = \phi_{ZC} \pm \phi$ , which is referred to as the abrupt current reversal effect, described in [14, 15].

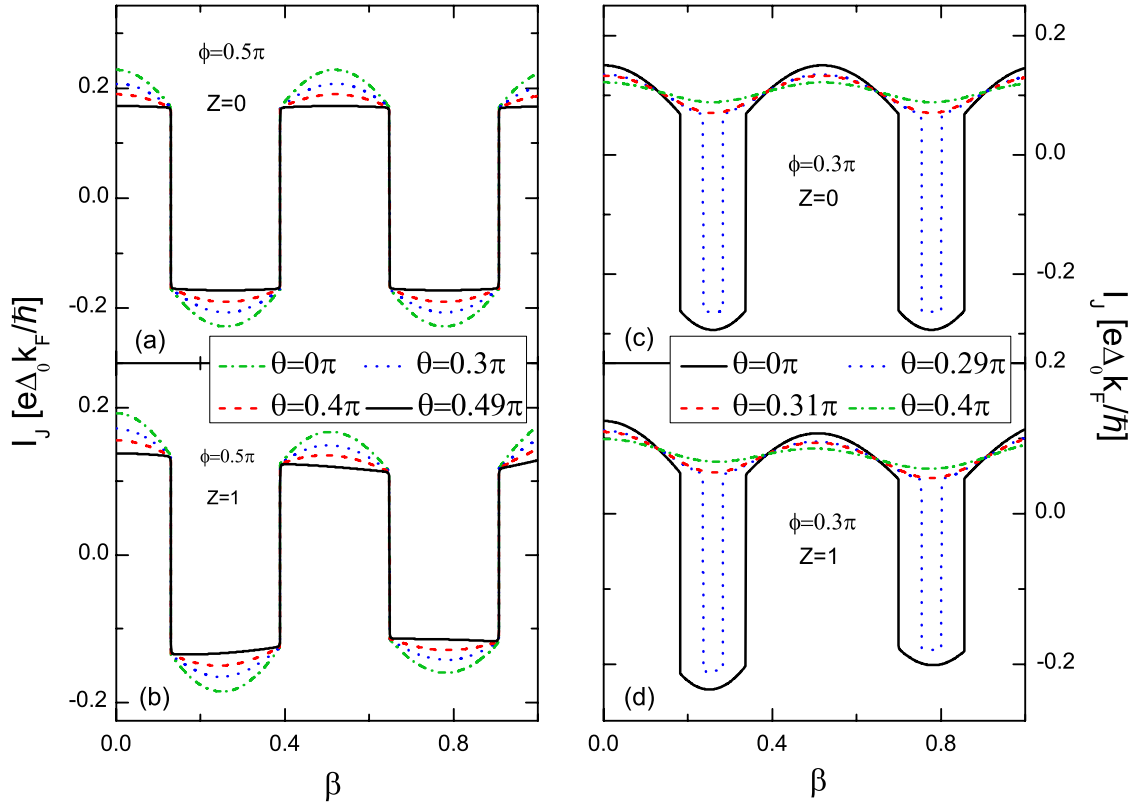
The RSOC is now turned on in the 2DEG region,  $\beta \neq 0$ . The current-phase relation  $I_J(\phi)$  is shown in figure 3 with different lengths  $L$  of the 2DEG between the two TSs. It is seen that RSOC can have an effect on  $I_J$ , and the discontinuous jump of supercurrent  $I_J$  is shown to be split into two steps at  $\phi_{ZC} \pm \gamma$  with  $\gamma = k_R L$  being the Rashba spin precession phase. When the spin-up and spin-down TS Cooper pairs enter the RSOC region, the pseudomagnetic field from RSOC (perpendicular to the momentum direction of the quasiparticle  $\mathbf{k}$ ) will cause them to precess so that the spin-up and spin-down space of



**Figure 3.** The  $\phi$  dependence of the Josephson current for  $\theta = 0$ ,  $\beta = 0.05$  with  $L = 20$  nm (a) and  $L = 30$  nm (b).

the TS order parameter are mixed. Therefore, the tunneling TS Cooper pairs may acquire such a precession phase, and the supercurrent exhibits a dependence on the phase  $\gamma$ . This is basically similar to the FM effect found in a very recent work by Brydon and Manske [28]. When an FM noncollinear to the spin direction of the TS order parameter is considered between the two TSs, the spin flipping effect could mix the tunneling spin-up and spin-down Cooper pairs, and these spin-flipped Cooper pairs can acquire an additional phase shift, similar to  $\gamma$  in this work. In figure 3(b), two discontinuous jumps will depart a little further from  $\phi_{ZC}$  with an increase of  $L$ , since  $\gamma$  is proportional the length  $L$  of the 2DEG. It is noted that the spin precession phase  $\gamma$  from RSOC is essentially different from the cross angle  $\theta$ , and it keeps time reversal symmetry so that there is no charge supercurrent, as  $\phi = 0$ , or spin supercurrent, as  $\theta = 0$ . In other words, RSOC can not lead to spin splitting of charge current by itself, especially for the 1D case studied here. When the interface barriers are considered  $Z \neq 0$ , the supercurrent is depressed a little in figure 3 and the discontinuous jump remains unchanged, which is different from the conventional s-wave supercurrent and comes from edge states forming at the interfaces.

We proceed by examining the Josephson current dependence on RSOC. The current  $I_J$  is plotted in figure 4 as a function of  $\beta$  by setting  $\phi = 0.5\pi$  with  $\theta \in \{0\pi, 0.3\pi, 0.4\pi, 0.49\pi\}$  in the left two panels and  $\phi = 0.3\pi$  with  $\theta \in \{0\pi, 0.29\pi, 0.31\pi, 0.4\pi\}$  in the right panels. It is shown in figures 4(a) and (b) that the charge current at  $\phi = 0.5\pi$  exhibits a periodic oscillation and remains antisymmetric around the current zeroth  $I_J = 0$ . The  $I_J$ - $\beta$  relation here resembles the  $I_J$ - $\theta$  relation in figure 2(b). An abrupt current



**Figure 4.** The  $\beta$  dependence of the Josephson current for  $\phi = 0.5\pi$  with  $\theta \in \{0\pi, 0.3\pi, 0.4\pi, 0.49\pi\}$  in the two left panels and  $\phi = 0.3\pi$  with  $\theta \in \{0\pi, 0.29\pi, 0.31\pi, 0.4\pi\}$  in the two right panels,  $Z = 0$  in (a), (c) and  $Z = 1$  in (b), (d),  $L = 20$  nm.

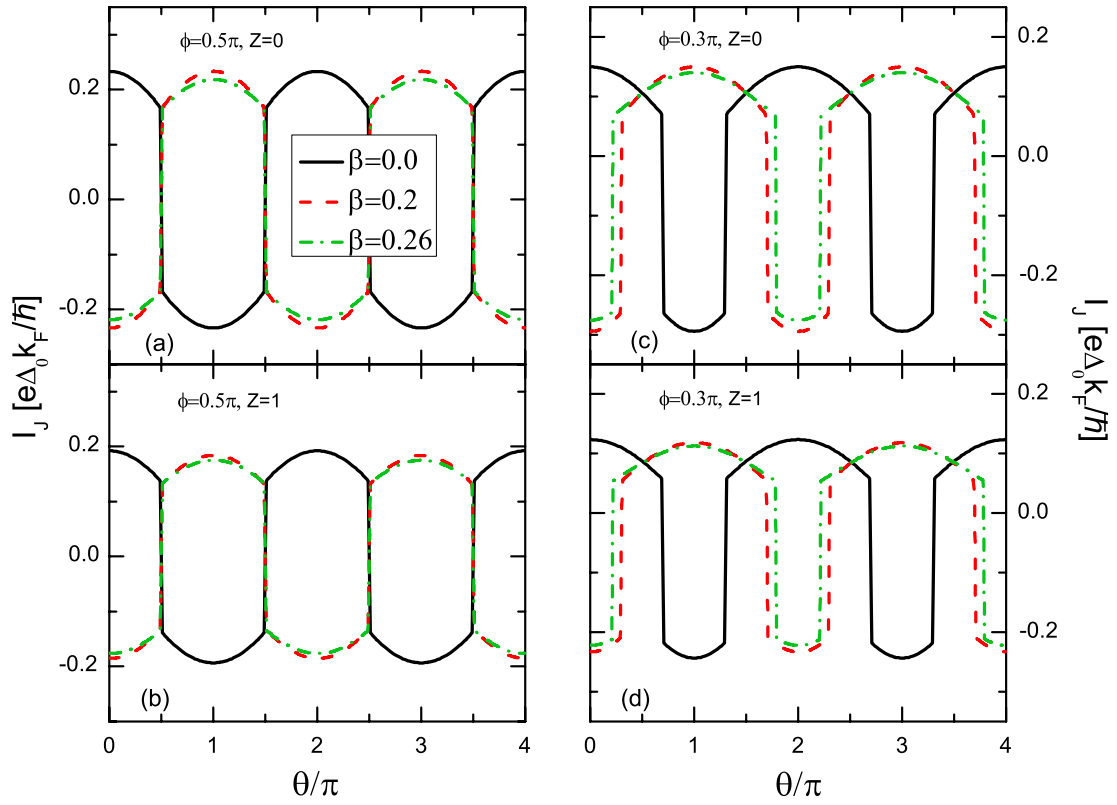
reversal effect appears, and the  $0-\pi$  transition occurs in this p-wave SC junction. It is also seen at  $\phi = 0.5\pi$  that the cross angle  $\theta$  has little influence on the  $I_J-\beta$  relation in figures 4(a) and (b), which suggests the spin precession phase  $\gamma$  from RSOC is not simply shifting  $\theta$  and depends crucially on the macroscopic phase  $\phi$ . When  $\theta = 0.5\pi$ , the current curve is a perfect rectangular wave at zero barrier strength  $Z = 0$ . In figures 4(c) and (d) with  $\phi = 0.3\pi$  fixed, the  $I_J-\beta$  is very different from those in figures 4(a) and (b). The perfect oscillation of  $I_J$  around the current zero is destroyed, and the negative value of current vanishes as  $|\theta - n\pi| > \phi$ , i.e., a discontinuous jump of the current does not appear, either, and the  $I_J-\beta$  exhibits the simple behavior of a cosine function.

As stated above, the discontinuous jump in  $I_J - \phi$  is at  $\phi = \phi_{ZC}$  when  $\theta = 0$  and  $\beta = 0$ , otherwise, the jump will split into two steps at  $\phi_{ZC} \pm \theta$ . In figures 5(a) and (b), the  $I_J-\theta$  curves are shown at  $\phi = 0.5\pi$ . The current jump occurs at  $\theta_c = m\pi$ , with  $m$  being a half integer, and RSOC does not shift these discontinuous points, which are consistent with those shown in figures 4(a) and (b). When  $\phi = 0.3\pi$ , the case is very different. The discontinuous jump  $\theta_c$  occurs at  $\theta_c = n\pi \pm \phi$  with  $\beta = 0$ , as shown in figures 5(c) and (d) (solid line). When the RSOC is turned on (dot-dash and dashed line), the discontinuous points  $\theta_c$  will change and RSOC can have an effect. Nevertheless, the current curves with different  $\beta$  cross each other at  $\theta = m\pi$ . It is also seen in figures 5(c) and (d) that at  $|\theta - n\pi| < \phi$ , the sign of  $I_J$  with  $\beta = 0$  can be opposite to those with  $\beta = 0.2$  and  $\beta = 0.29$ , whereas at  $|\theta - n\pi| > \phi$ , the signs of  $I_J$  for all three values of  $\beta = 0, 0.2, 0.29$  are the same,  $I_J > 0$ . This

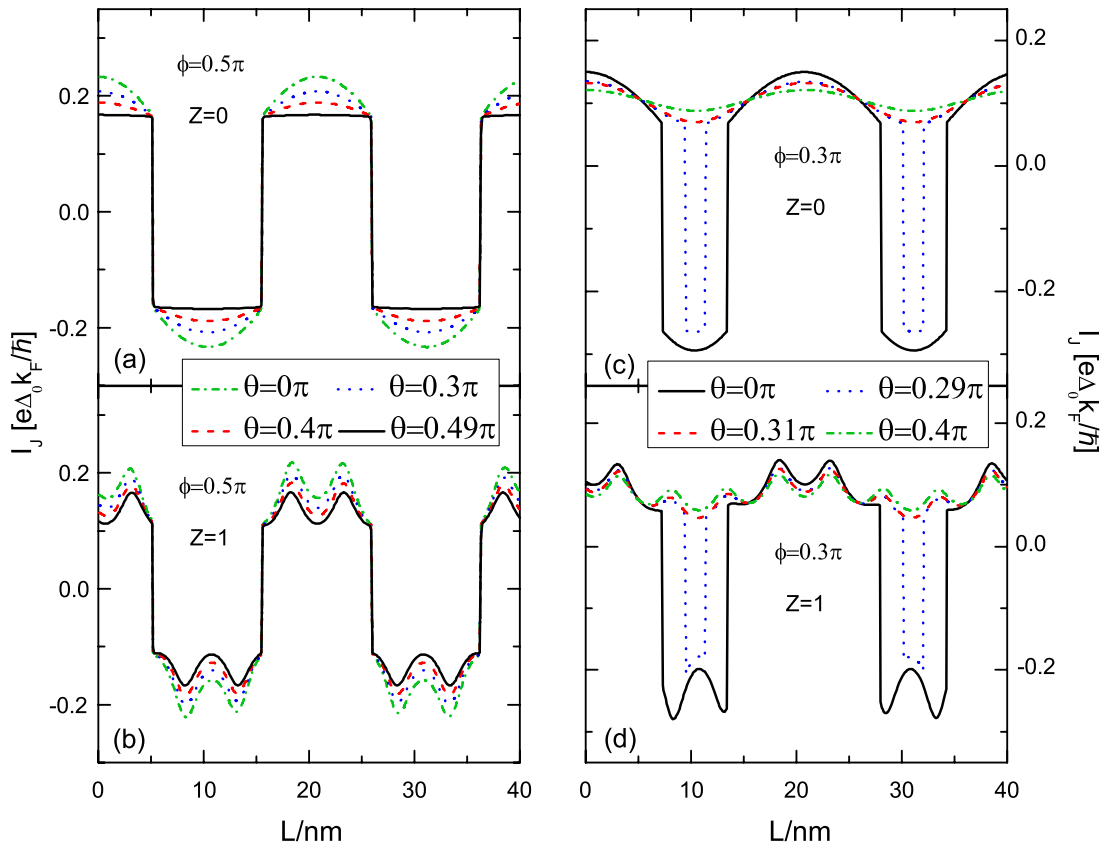
is consistent with the curves in figures 4(c) and (d) that  $I_J$  can only be a simple cosine function when  $|\theta - n\pi| > \phi$ . It is believed that due to the maximum of the cross angle  $\theta$  between two  $\mathbf{d}$  being  $\pm 0.5\pi$ , the spin precession phase  $\gamma$  cannot cause any additional shift phase as the compensated  $\theta_c = \pm 0.5\pi$  with  $\phi = 0.5\pi$ .

Since the spin precession phase  $\gamma$  is determined by both the length  $L$  of the RSOC region and the strength of RSOC, the Josephson current is also expected to be modulated by the length  $L$ . The results are presented in figure 6.  $I_J$  exhibits a perfect oscillation with  $L$ , and the whole profile of the curves resemble those in figure 4 where the  $I - \beta$  relations are shown. For the  $Z = 0$  case, a supercurrent of the rectangular wave type also appears with a variation of  $L$  as  $\phi = \theta = 0.5\pi$  (figure 6(a), black-solid line). In addition to oscillations of the supercurrent due to the spin precession phase, there exists another type of oscillation at  $Z \neq 0$  with a smaller period, shown in figures 6(b) and (d), which stems from the usual resonant tunneling of quasiparticles in the RSOC region due to the two interface barriers. The oscillation period is related to the Fermi wavevector  $k_F$ , since  $k_R/k_F = 0.25$  at  $\beta = 0.5$ . The larger period from RSOC is exactly four times the smaller one from resonant tunneling, as shown in figures 6(b) and (d).

The properties of the supercurrent in the TS/2DEG/TS junction shown above indicate that RSOC can indeed make a difference on the p-wave Josephson current, and the  $0-\pi$  transition can be realized; unlike the s-wave Josephson current, where RSOC was shown to have no influence on the supercurrent, especially in the 1D case. Since the



**Figure 5.** The dependence of the Josephson current on  $\theta$  for  $\phi = 0.5\pi$  in the two left panels and  $\phi = 0.3\pi$  in the two right panels with  $L = 20$  nm and  $\beta \in \{0, 0.2, 0.26\}$ ,  $Z = 0$  for (a) and (c),  $Z = 1$  for (b), (d).



**Figure 6.** The  $L$  dependence of the Josephson current for  $\phi = 0.5\pi$  with  $\theta \in \{0\pi, 0.3\pi, 0.4\pi, 0.49\pi\}$  in the two left panels and  $\phi = 0.3\pi$  with  $\theta \in \{0\pi, 0.29\pi, 0.31\pi, 0.4\pi\}$  in the two right panels,  $Z = 0$  in (a), (c) and  $Z = 1$  in (b), (d),  $\beta = 0.5$ .

RSOC strength was demonstrated to be controlled by a perpendicular electric field upon the 2DEG plane [20], our findings in this work are measurable with today's experimental techniques. More importantly, the abrupt current reversal effect in TS/2DEG/TS, found in [14, 15], can also be realized by modulating the RSOC via an electric field. This is much more convenient than either modulating the magnitude and direction of an FM moment, or rotating the cross angle  $\theta$  of the two  $\mathbf{d}$  vectors that point in a fixed direction in crystal.

#### 4. Conclusions

In summary, we have investigated the RSOC effect on the supercurrent in a clean TS/2DEG/TS junction by using the BdG equation and the quantum scattering method. Unlike the singlet order parameter of a s-wave SC, the TS order parameter with equal spin pairing can be influenced by RSOC, and the tunneling Cooper pairs in the RSOC region can achieve an additional phase, the spin precession phase. Therefore, the supercurrent exhibits an oscillating behavior and the  $0-\pi$  transition occurs by modulating the RSOC strength. In addition, the current switch effect can also be found in this junction by controlling the RSOC instead of modulating the  $\mathbf{d}$  vector directions or the FM moment. Our results may shed light on the modulation of the Josephson current by purely electric means without involving any magnetic factors.

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